

Warm-up!

Multiply the following matrices (by hand).

$$\begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 10 & 20 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 3 \\ 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 23 \\ 4 \end{bmatrix}$$

4.8 Using Matrices to Solve Systems of Equations

Matrix Equation

$$\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

coeff. matrix variable matrix constant matrix

$$\begin{bmatrix} 1x + 3y \\ 1x + 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\begin{cases} x + 3y = 3 \\ x + 2y = 7 \end{cases}$$

Matrix Equation

$$\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

Coefficient Matrix Variable Matrix Constant Matrix

Put the following system into a matrix equation:

$$5x - 3y = 9$$

$$2x + 3y = 7$$

$$\begin{bmatrix} 5 & -3 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$$

Use Matrix equation to solve

A- coefficient matrix
X- variable matrix
B- constant matrix

$$a x = b$$

$$\begin{aligned}\frac{1}{a} \cdot a x &= \frac{1}{a} b \\ 1 x &= \frac{1}{a} b \\ x &= \frac{1}{a} b\end{aligned}$$

$$A X = B$$

$$\begin{aligned}A^{-1} \cdot A X &= A^{-1} \cdot B \\ I \cdot X &= A^{-1} \cdot B \\ X &= A^{-1} B\end{aligned}$$

ex:

$$5x + 3y = 13$$

$$4x + 7y = -8$$

$$(5, -4)$$

$$\begin{bmatrix} 5 & 3 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 4 & 7 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 13 \\ -8 \end{bmatrix}$$

$$\frac{9L}{23} + \frac{24}{23}$$

$$\frac{115}{23} = 5$$

$$\frac{1}{23} \begin{bmatrix} 7 & -3 \\ -4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 13 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} \frac{7}{23} & -\frac{3}{23} \\ -\frac{4}{23} & \frac{5}{23} \end{bmatrix} \cdot \begin{bmatrix} 13 \\ -8 \end{bmatrix}$$

ex:

$$5x - 2y = 3$$

$$8x + 4y = 3$$

$$\left(\frac{1}{2}, -\frac{1}{4}\right)$$

$$\begin{bmatrix} 5 & -2 \\ 8 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 8 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

ex:

$$10x + 5y = 15$$

$$6x + 3y = -6$$

$$\begin{aligned}10x + 5y &= 15 \rightarrow 2x + y = 3 \\ 6x + 3y &= -6 \rightarrow 2x + y = -3\end{aligned}$$

$$\begin{bmatrix} 10 & 5 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -6 \end{bmatrix}$$

Determine if it's ∞ or \emptyset

Do:

$$\begin{aligned} 1. \quad & 2x + 3y = 10 \\ & x + 6y = 32 \end{aligned}$$

$$\begin{aligned} 2. \quad & 3x + 4y = -1 \\ & 6x - 2y = 3 \end{aligned}$$

ex:

$$3x - 2y + z = 0$$

$$2x + 3y - z = 17$$

$$5x - y + 4z = -7$$

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 5 & -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 17 \\ -7 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$$

HW

p205-206

4-6, 24, 26-28